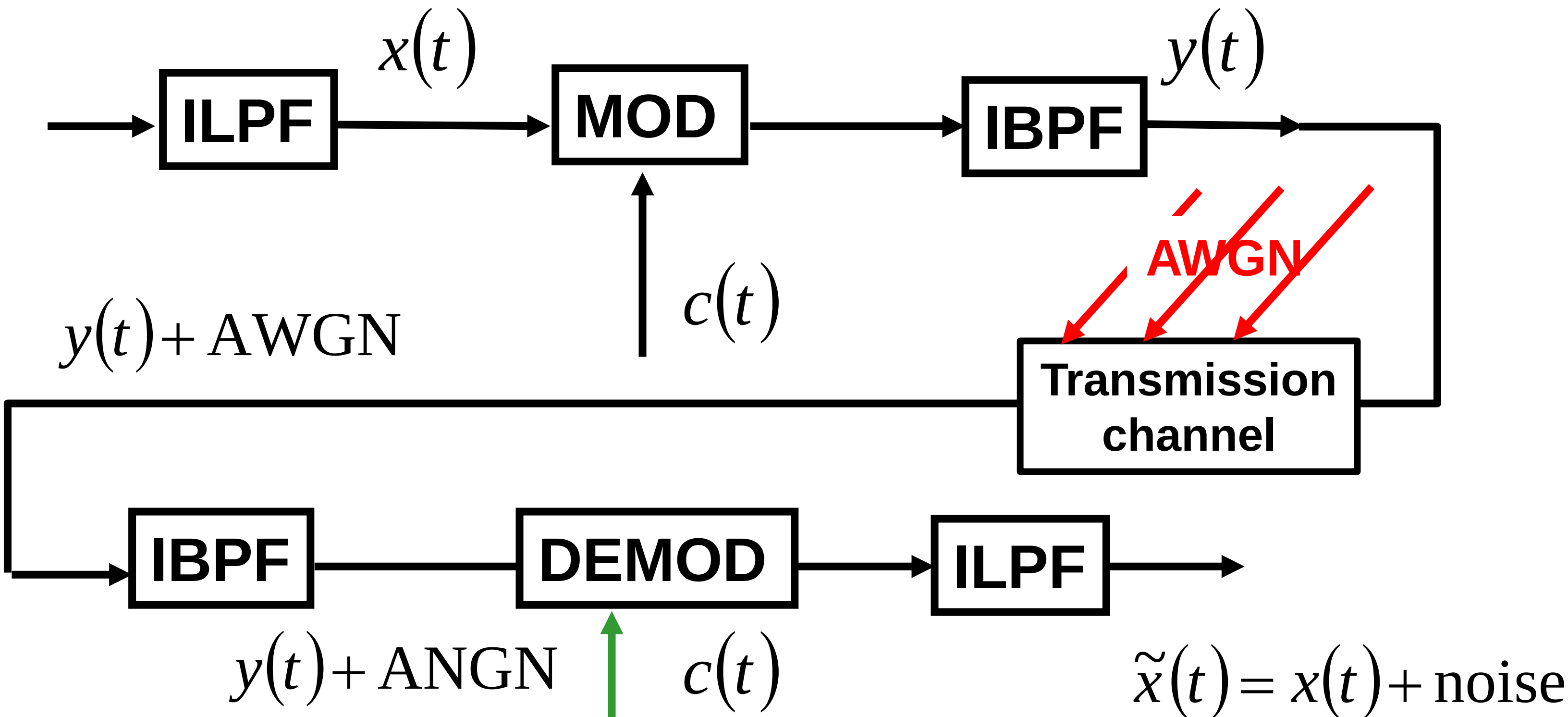
AM and FM Noise Immunity (10)

- Telecommunication system with noisy transmission channel:
  - AM – envelope detection
  - FM – detection by phase angle detection and differentiation
- Summary

Transmission System with Noise

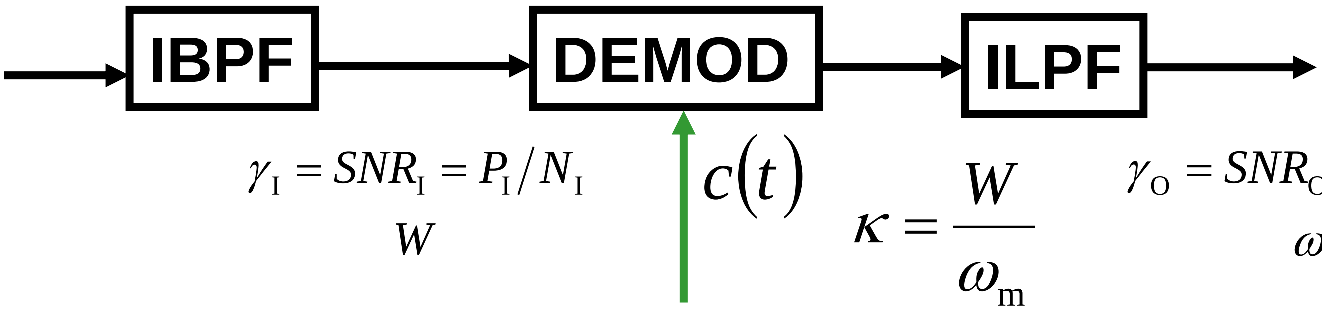


$$\gamma_{\mathrm{I}} = SNR_{\mathrm{I}} \qquad \gamma_{\mathrm{O}} = SNR_{\mathrm{O}}$$

$$SNR = (\text{signal power})/(\text{noise power}) = S/N \ [\text{dB}]$$

Transmission System Noise Immunity

$y(t) + \mathrm{AWGN}(\eta)$ $\quad$ $y(t) + \mathrm{ANGN}(\eta)$ $\quad$ $\tilde{x}(t) = x(t) + \text{noise}$



$\gamma_{\mathrm{I}} = SNR_{\mathrm{I}} = P_{\mathrm{I}} / N_{\mathrm{I}}$

W

$\kappa = \dfrac{W}{\omega_{\mathrm{m}}}$

$\gamma_{\mathrm{O}} = SNR_{\mathrm{O}} = P_{\mathrm{O}} / N_{\mathrm{O}}$

ω_{m}

$$\text{modulation gain} \quad g = \frac{SNR_{\mathrm{O}}}{SNR_{\mathrm{I}} \times \kappa} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}} \times \kappa} = \frac{\gamma_{\mathrm{O}}}{\gamma}$$

noise characteristic

$$\gamma_{\mathrm{I}} \times \kappa = \frac{P_{\mathrm{I}}}{N_{\mathrm{I}}} \times \frac{W}{\omega_m} = \frac{P_{\mathrm{I}}}{\eta W / \pi} \times \frac{W}{\omega_{\mathrm{m}}} = \frac{P_{\mathrm{I}}}{\eta \omega_{\mathrm{m}} / \pi} = \gamma$$

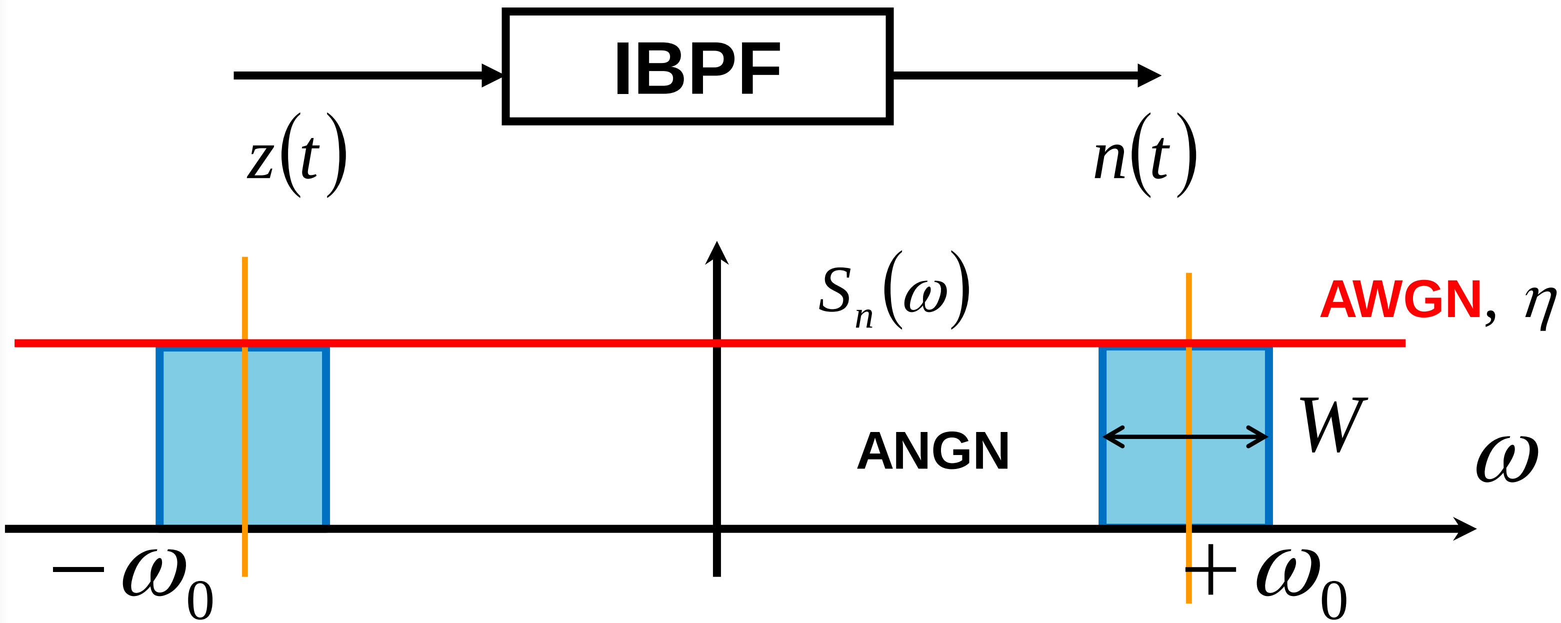
$$\gamma_{\mathrm{O}} = g \times \gamma$$

$$\gamma_{\mathrm{O}}[\mathrm{dB}] = 10 \log g[\mathrm{dB}] + \gamma[\mathrm{dB}]$$

ANGN - Lowpass Representation

Additive Wideband Gaussian Noise (AWGN)

Additive Narrowband Gaussian Noise (ANGN)

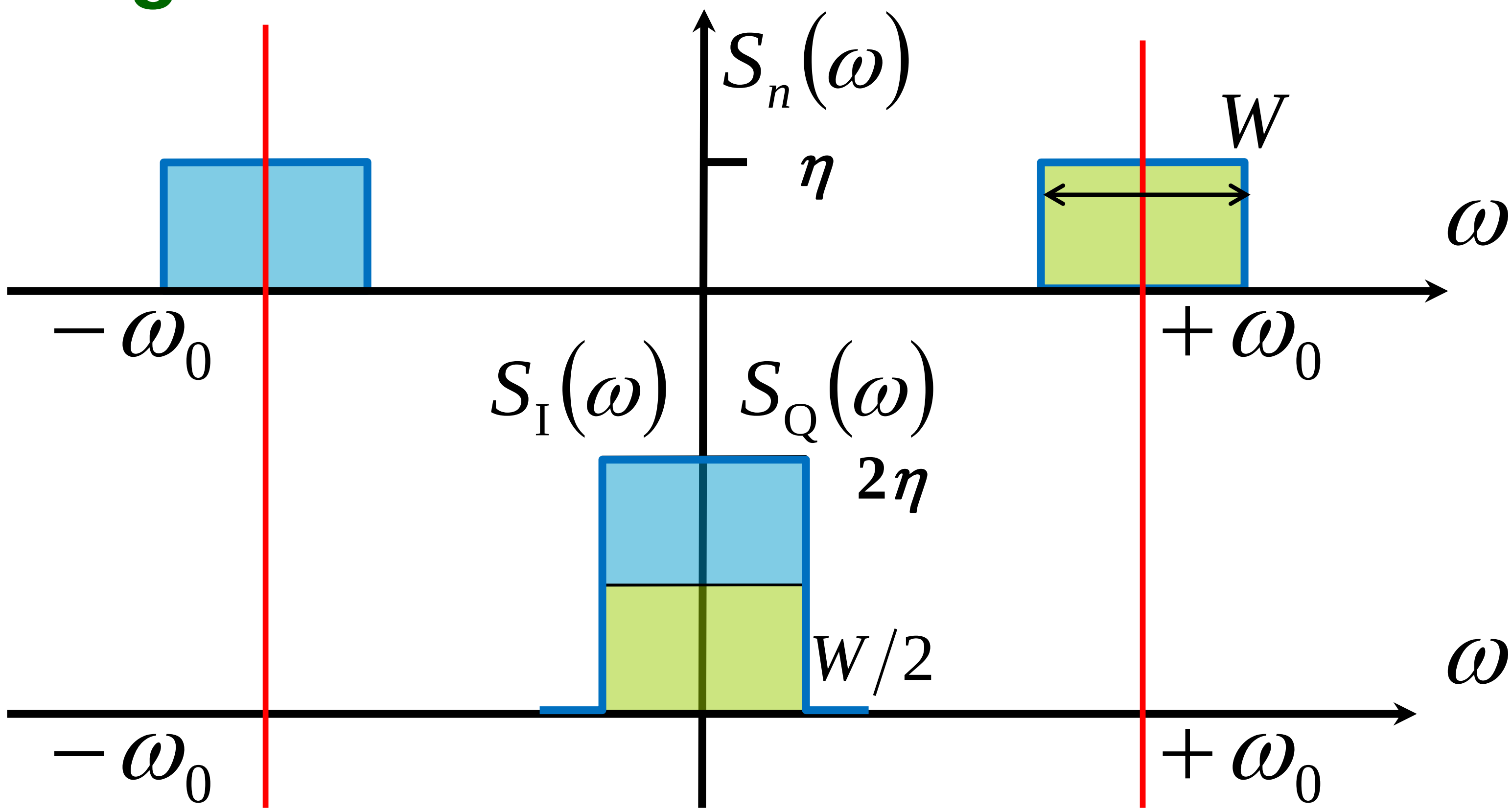


Lowpass representation

$$n(t) = n_{\mathrm{I}}(t)\cos\omega_0 t - n_{\mathrm{Q}}(t)\sin\omega_0 t$$

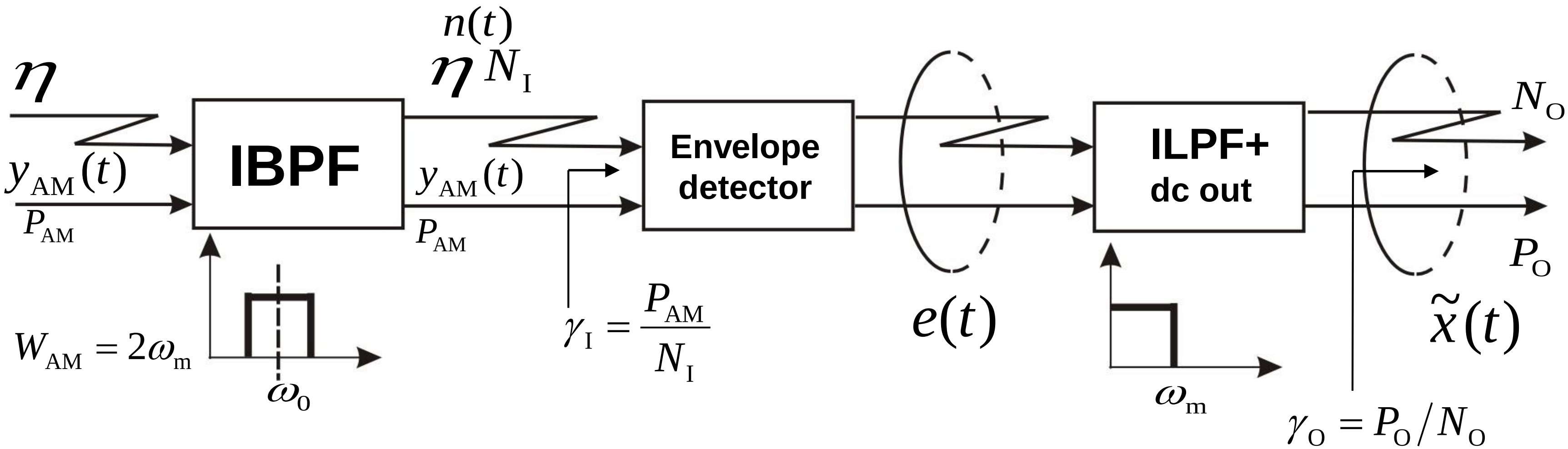
Lowpass representation of the ANGN consists of two lowpass noise components (inphase comp. $n_{\mathrm{I}}(t)$, quadrature comp. $n_{\mathrm{Q}}(t)$) modulating carriers in quadrature.

ANGN – double sideband case – power budget



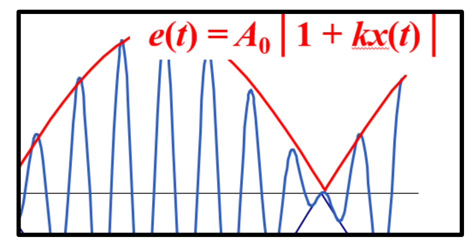
Lowpass representation allows for analyzing the detection proces in the lowpass bandwidth (in the modulating signal bandwidth).

AM system – envelope detection



$$y_{AM}(t) = A_0[1 + kx(t)]\cos\omega_0 t$$

$$e(t) = \left|A_0[1 + kx(t)]\right| = A_0[1 + kx(t)],\ 1 + kx(t) \geq 0$$



$$y_{AM}(t) + n(t) = A_0[1 + kx(t)]\cos\omega_0 t + n_I(t)\cos\omega_0 t - n_Q(t)\sin\omega_0 t$$

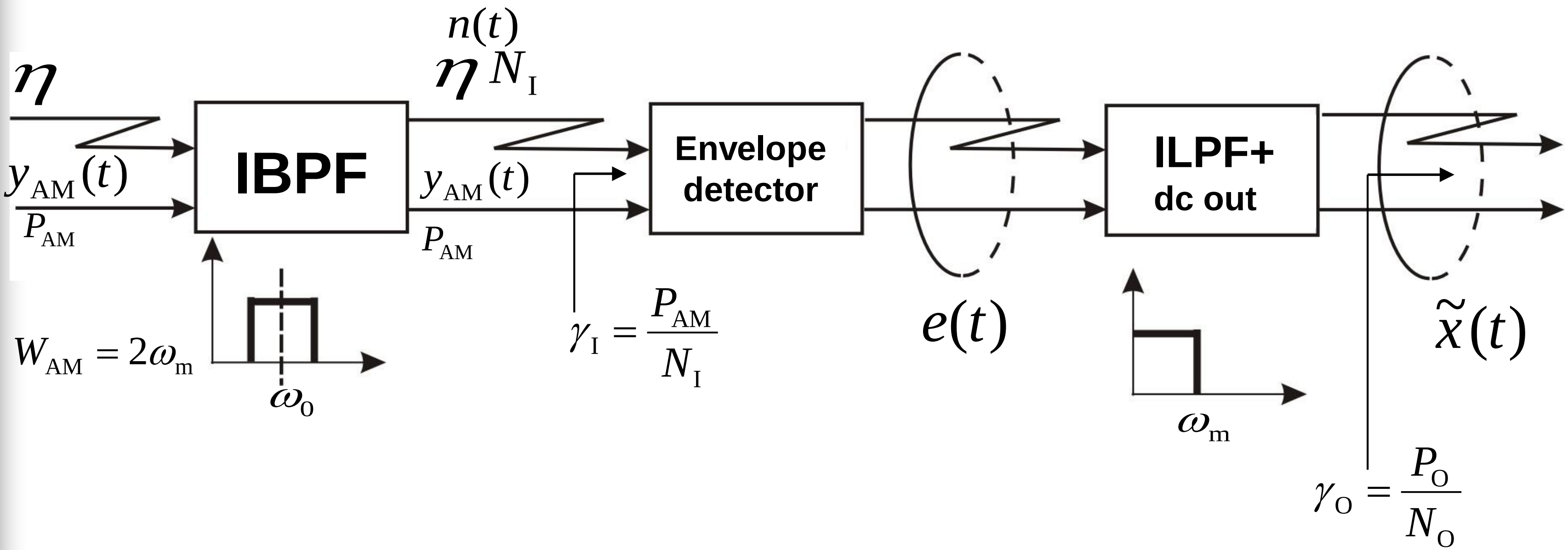
$$y_{AM}(t) + n(t) = \sqrt{[A_0 + kA_0 x(t) + n_I(t)]^2 + n_Q^2(t)}\,\cos[\omega_0 t + \varphi(t)]$$

$$e(t) = \sqrt{[A_0 + kA_0 x(t) + n_I(t)]^2 + n_Q^2(t)}$$

How much is

$\tilde{x}(t) = x(t) + \text{noise}$

AM system - envelope detection



$$e(t) = \sqrt{[A_0 + kA_0 x(t) + n_I(t)]^2 + n_Q^2(t)}$$
$$\tilde{x}(t) = \mathrm{ILPF}[e(t)]$$

During the envelope detection of the AM signal the information signal and lowpass noise components get mixed in a highly nonlinear way.

Overthreshold Envelope Detection (signal dominates noise)

$$e(t)=\sqrt{\left[A_0+kA_0x(t)+n_{\mathrm{I}}(t)\right]^2+n_{\mathrm{Q}}^2(t)}=A_0\sqrt{\left[1+kx(t)+\frac{n_{\mathrm{I}}(t)}{A_0}\right]^2+\left[\frac{n_{\mathrm{Q}}(t)}{A_0}\right]^2}$$

$$n_{\mathrm{I}}/A_0\,,n_{\mathrm{Q}}/A_0\approx 0$$

$$e(t)\approx A_0\left[1+kx(t)\right]+n_{\mathrm{I}}(t)$$

Receiver output signal (dc out):

$$\tilde{x}(t)=\mathrm{LPF}\left[e(t)\right]\cong kA_0x(t)+n_{\mathrm{I}}(t)$$

Linear approximation – Maclaurin power series:

$$f(x,y)\approx f(0,0)+f_x'(0,0)x+f_y'(0,0)y,\ldots x,y\approx 0$$

$$f(x,y;t)=e\left(x=n_{\mathrm{I}}/A_0\,,y=n_{\mathrm{Q}}/A_0;t\right)$$

Modulation Gain – AM

$$g_{\mathrm{AM}} = \frac{\gamma_{\mathrm{O}}}{\kappa\gamma_{\mathrm{I}}} \qquad W_{\mathrm{AM}} = 2\omega_{\mathrm{m}} \qquad \kappa = W_{\mathrm{AM}}/\omega_{\mathrm{m}} = 2$$

$$\varphi_{\mathrm{AM}}(t) = A_0[1 + kx(t)]\cos\omega_0 t = A_0\cos\omega_0 t + kA_0 x(t)\cos\omega_0 t$$

$$\gamma_{\mathrm{I}} = P_{\mathrm{I}}/N_{\mathrm{I}} \qquad P_{\mathrm{I}} = P_{\mathrm{AM}} = \frac{1}{2}A_0^2 + \frac{1}{2}k^2 A_0^2 \overline{x^2} \qquad N_{\mathrm{I}} = \frac{\eta W_{\mathrm{AM}}}{\pi} = \frac{2\eta\omega_{\mathrm{m}}}{\pi}$$

$$\tilde{x}(t) \cong kA_0 x(t) + n_{\mathrm{I}}(t)$$

$$\gamma_{\mathrm{O}} = P_{\mathrm{O}}/N_{\mathrm{O}} \qquad P_{\mathrm{O}} = k^2 A_0^2 \overline{x^2} \qquad N_{\mathrm{O}} = \overline{n_{\mathrm{I}}^2} = \frac{2\eta\omega_{\mathrm{m}}}{\pi}$$

$$g_{\mathrm{AM}} = \frac{\gamma_{\mathrm{O}}}{\kappa\gamma_{\mathrm{I}}} = \frac{k^2\overline{x^2}}{1 + k^2\overline{x^2}} < 1$$

The AM system (envelope detection) provides the modulation loss.

Modulation loss – single tone AM

$$x(t) = a\cos\omega_{\mathrm{m}}t \qquad \overline{x^2} = \frac{1}{2}a^2$$

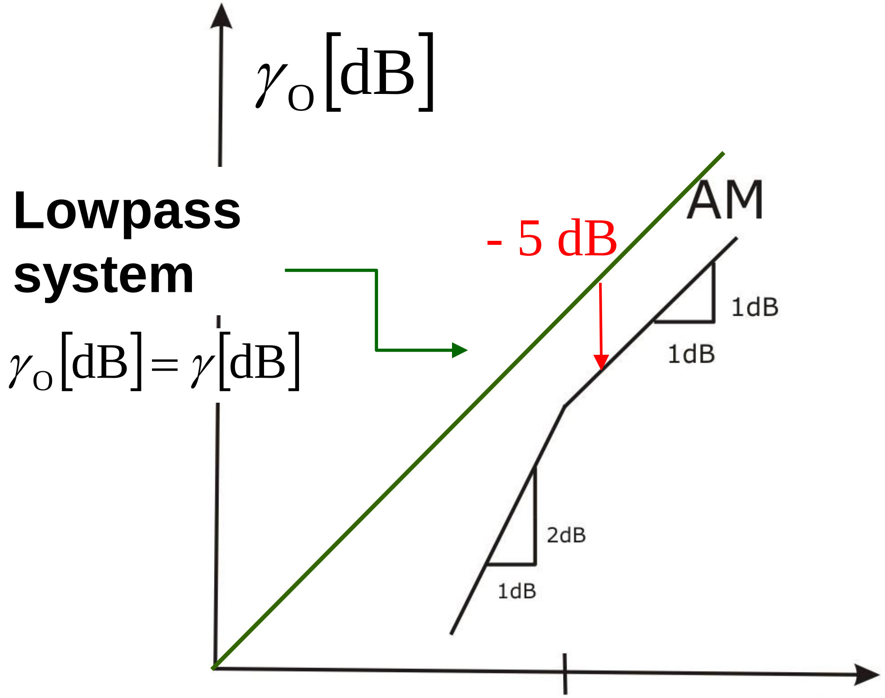
$$g_{\mathrm{AM}} = \frac{\gamma_{\mathrm{O}}}{\kappa\gamma_{\mathrm{I}}} = \frac{k^2\overline{x^2}}{1+k^2\overline{x^2}}$$

$$g_{\mathrm{AM}} = \frac{\frac{1}{2}k^2a^2}{1+\frac{1}{2}k^2a^2} = \frac{\frac{1}{2}m^2}{1+\frac{1}{2}m^2} = \frac{m^2}{2+m^2} \le \frac{1}{3} \approx -5\,\mathrm{dB}$$

The single tone AM system (envelope detection) provides the modulation loss.

AM Noise Characteristics

AM noise characteristic



$\gamma^* = 13\,\text{dB}$

$(\gamma_I^* = 10\,\text{dB})$

$$\gamma_O = g\gamma$$

$$\gamma_O[\text{dB}] = 10\log g[\text{dB}] + \gamma[\text{dB}]$$

$$\gamma_O[\text{dB}] = g_{AM}[\text{dB}] + \gamma[\text{dB}]$$

$$g_{AM}[\text{dB}] \le -5\,\text{dB} < 0$$

Overthreshold for $\gamma_I^* \ge 10\,\text{dB}$

(common agreement $P_I = 10 \times N_I$)

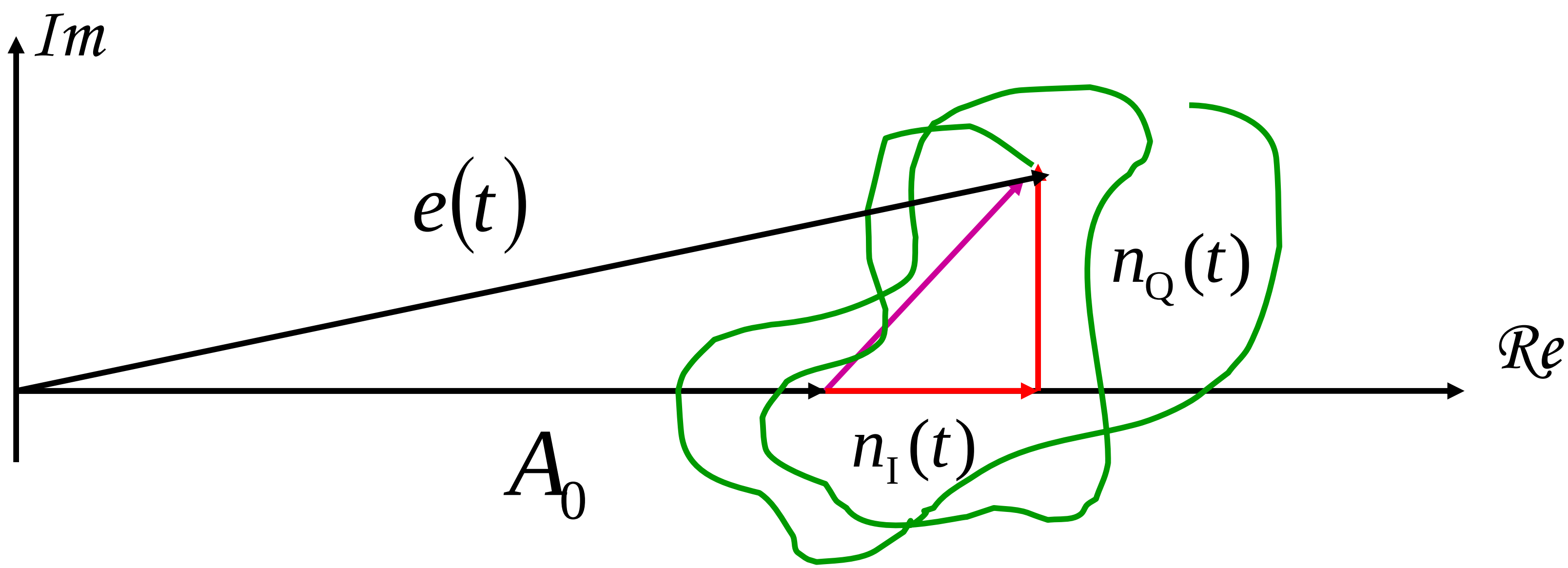
$$\gamma = \kappa\gamma_I = 2\gamma_I$$

$$\gamma^*[\text{dB}] = 10\log 2 + \gamma_I^*[\text{dB}] = 13\,\text{dB}$$

AM noise characteristic:

- **Threshold effect,**
- **Modulation loss,**
- **Tradeoff 1 : 1 (neutral) or 1 : 2 (unfavorable)**

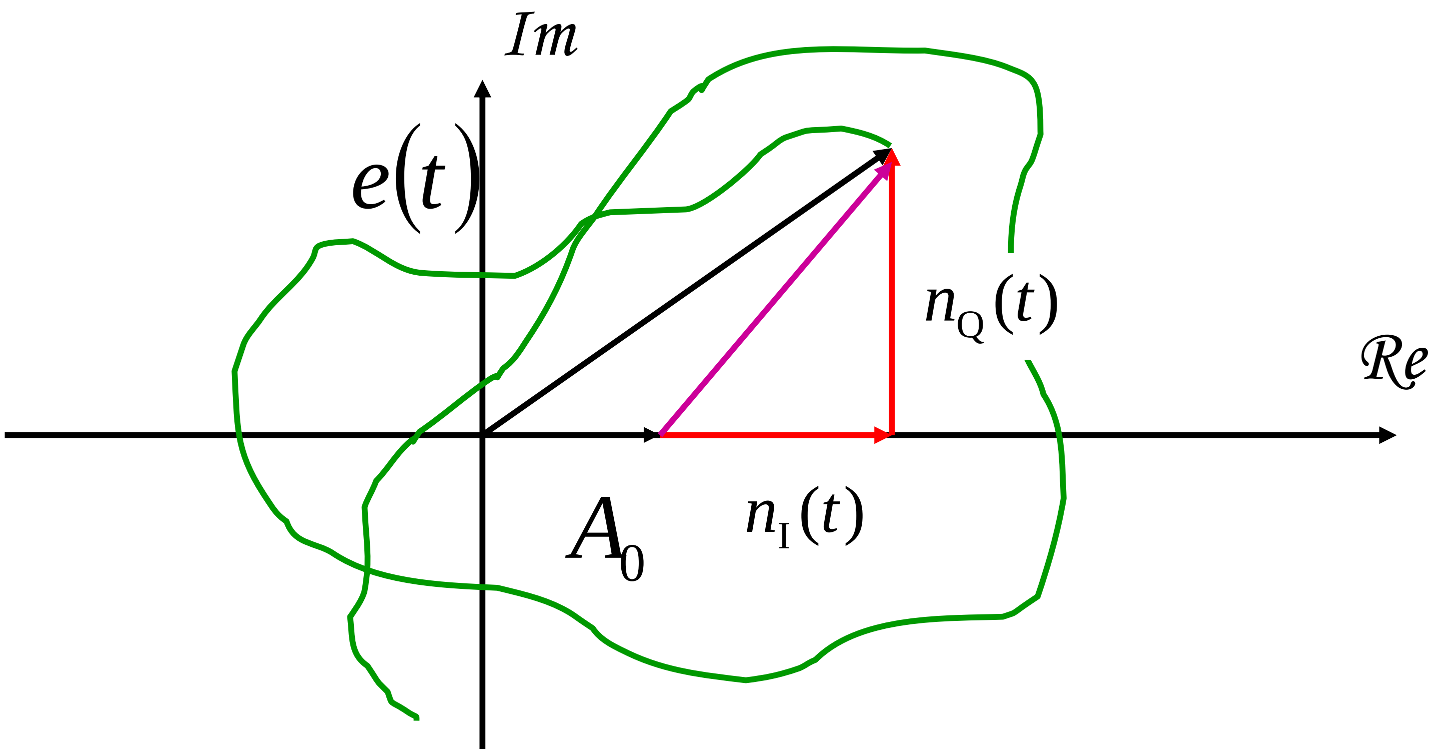
AM threshold effect
(overthreshold operation)
(unmodulated carrier transmission)



Signal dominates noise.

$$n_I, n_Q << A_0$$

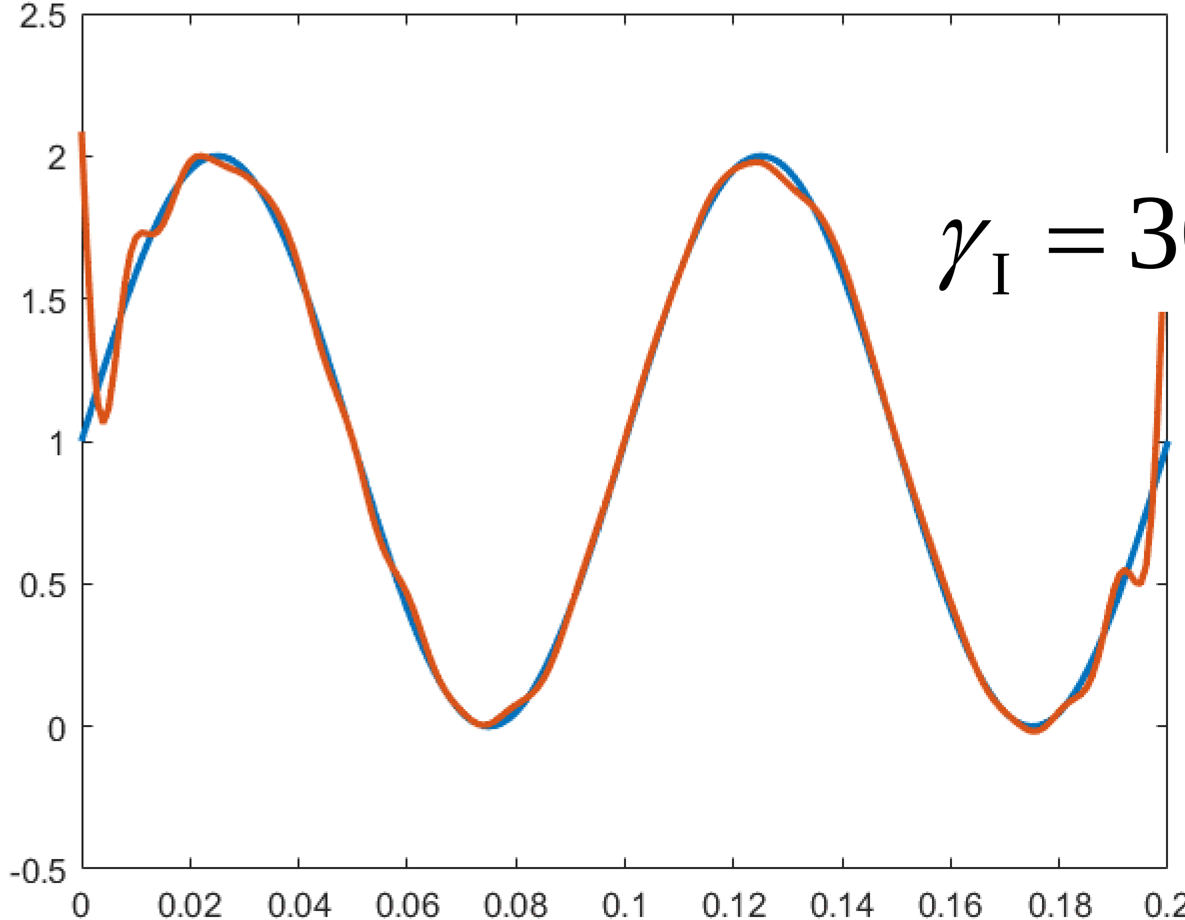
AM threshold effect
(subthreshold operation)
(unmodulated carrier transmission)



Noise captures the signal.

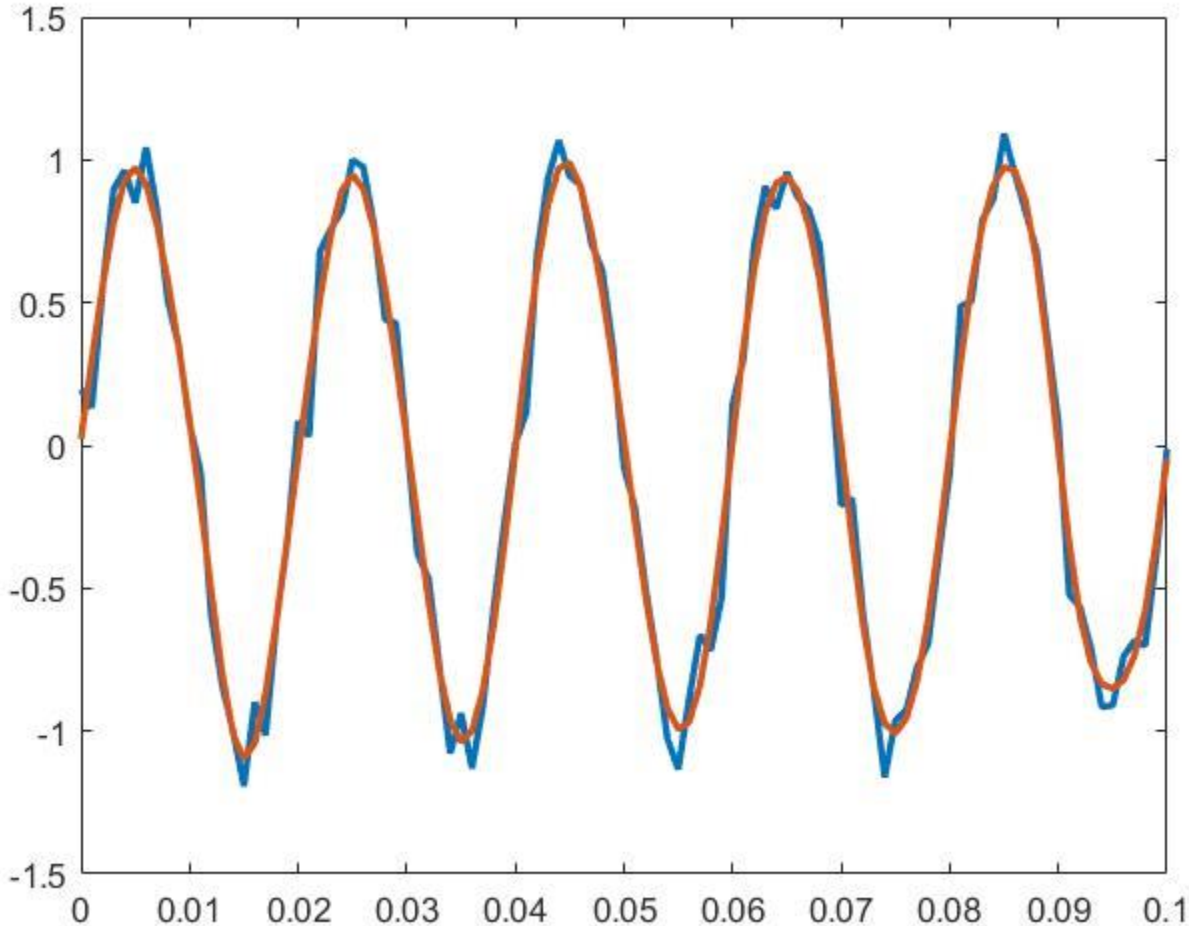
$$n_I, n_Q >> A_0$$

Envelope Detection – AM
Output signal



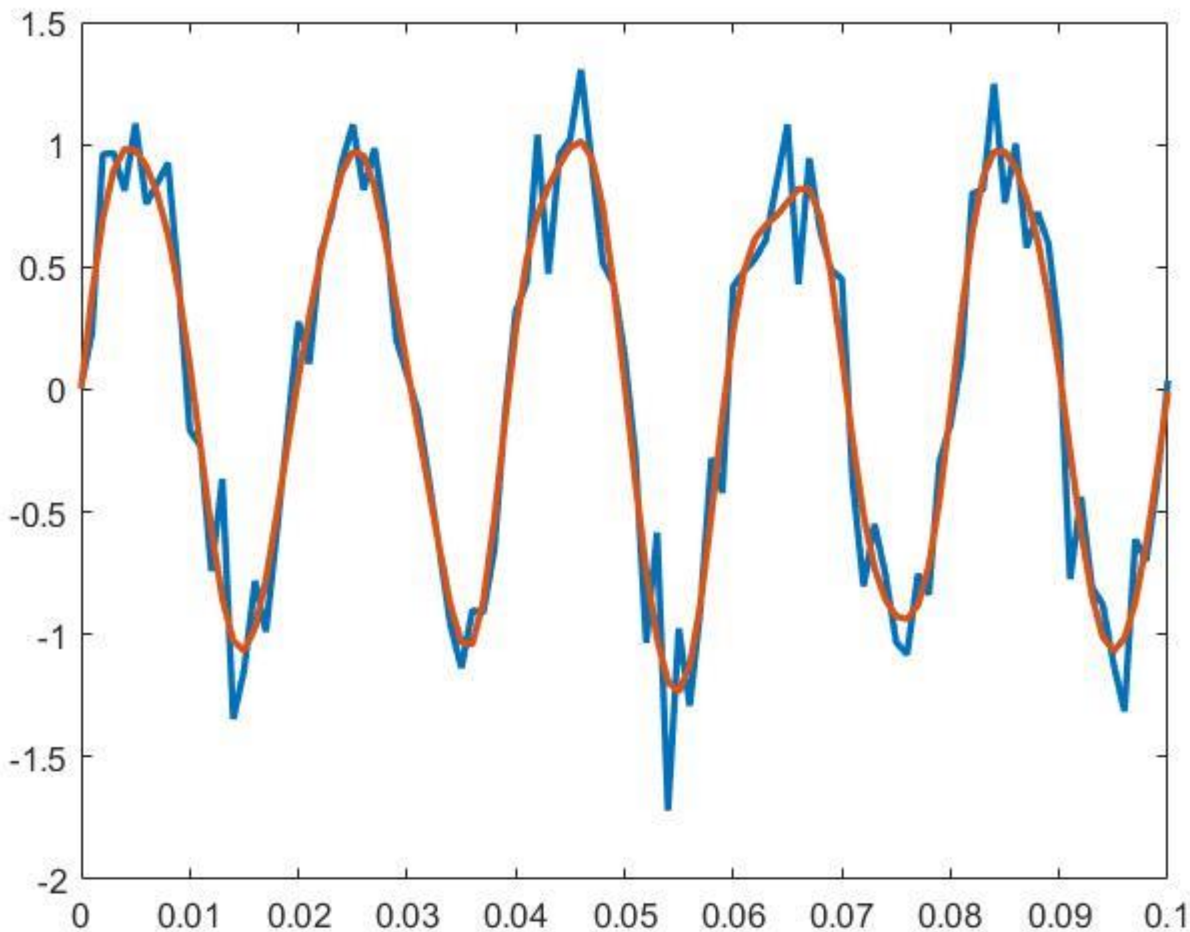
$$\gamma_{\mathrm{I}} = 30\,\mathrm{dB} > \gamma_{\mathrm{I}}^{*}$$

Envelope Detection – AM
Output signal



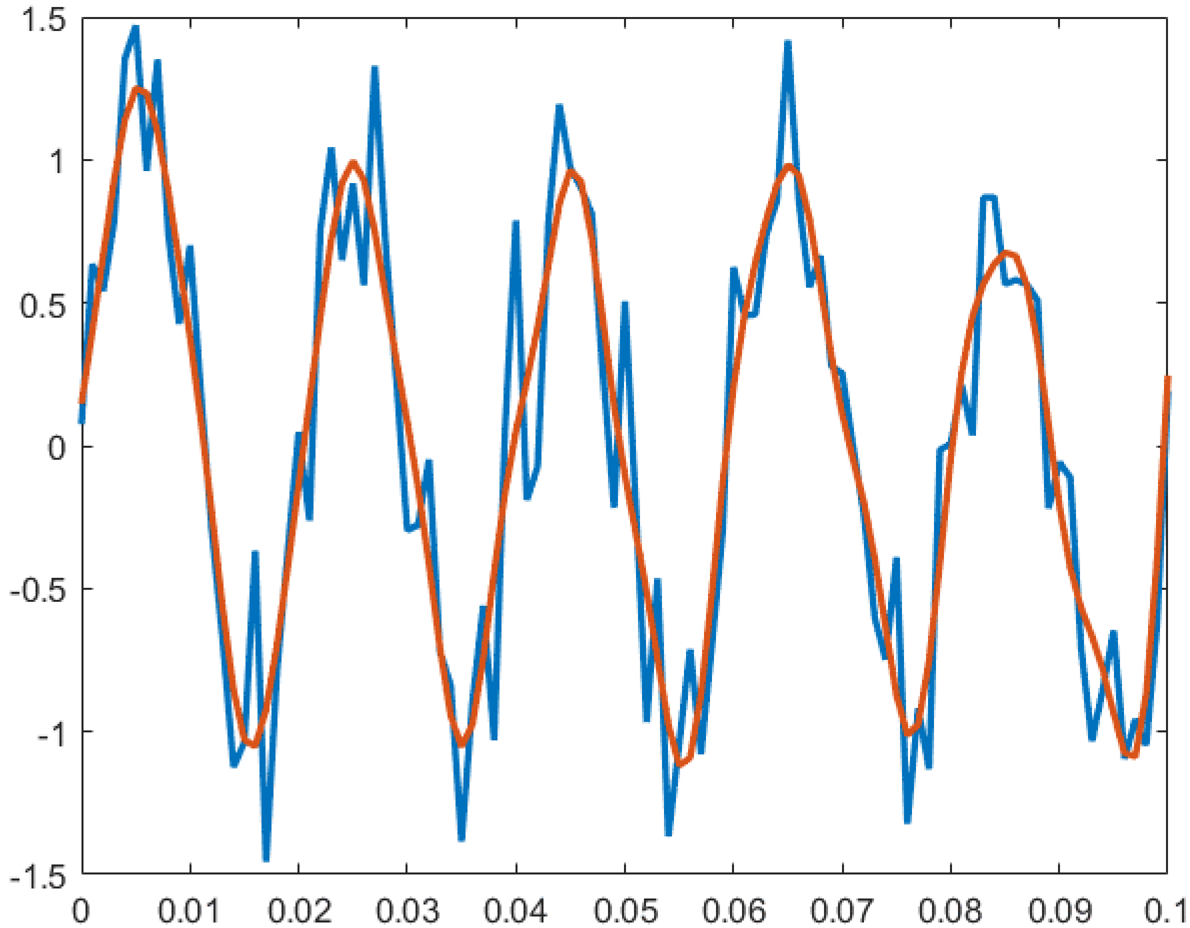
$$\gamma_{\mathrm{I}} = 20\,\mathrm{dB} > \gamma_{\mathrm{I}}^{*}$$

Envelope Detection – AM Output signal



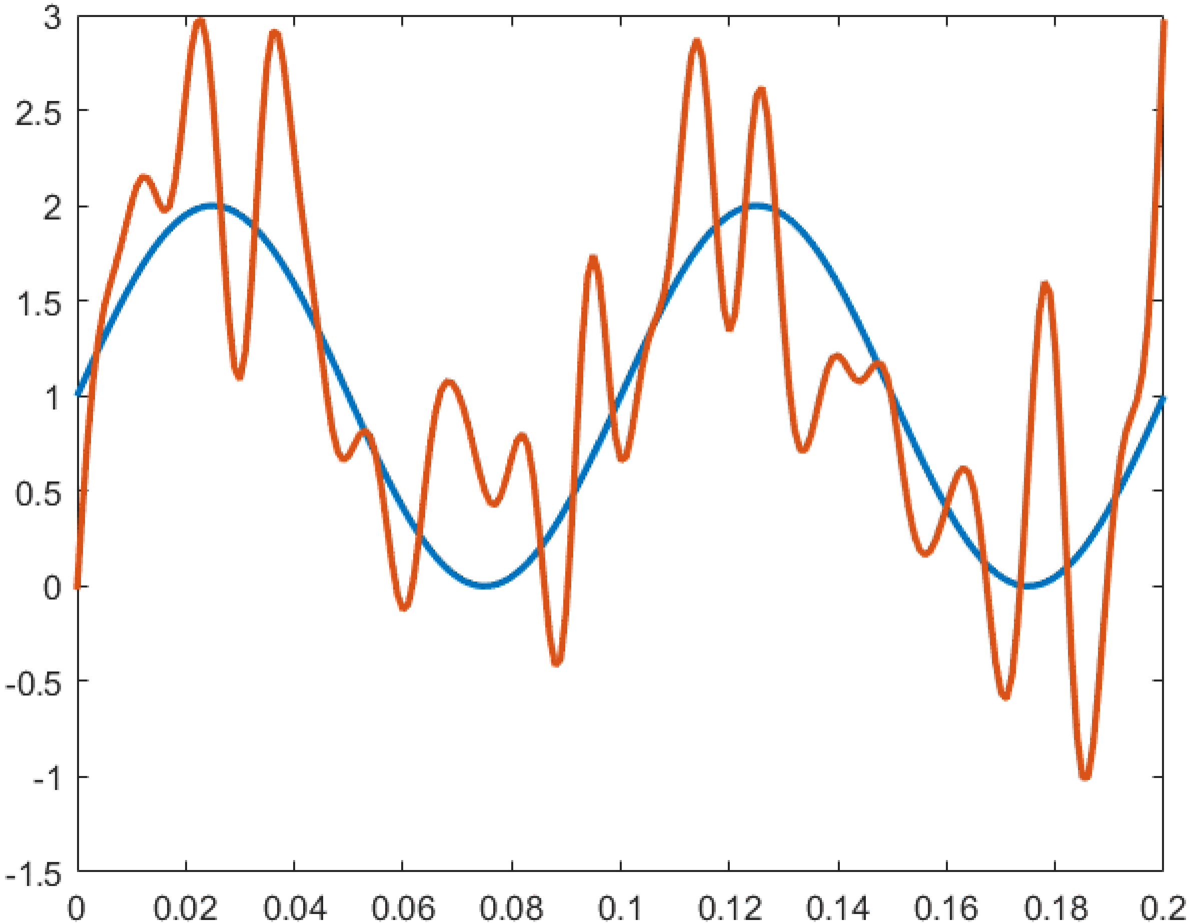
$$\gamma_{\mathrm{I}} = 13\,\mathrm{dB} > \gamma_{\mathrm{I}}^{*}$$

Envelope Detection – AM Output signal



$$\gamma_{\mathrm{I}} = \gamma_{\mathrm{I}}^{*} = 10\,\mathrm{dB}$$

Envelope Detection – AM Output signal



$$\gamma_{\mathrm{I}} = 0\,\mathrm{dB} < \gamma_{\mathrm{I}}^{*}$$

Coherent detection of the AM-DSB signal

Find the modulation gain and noise characteristic for a coherent detection of the AM-DSB signal $x(t)\cos\omega_0 t$ where $x(t)$ is a stationary random proces with the power $\overline{x^2}$.

Does the noise chracteristic of the coherently detected AM-DSB signal have the threshold?

Envelope detection - overthreshold operation - signal/noise separation

Prove that for the overthreshold operation the output signal of the envelope detector is a sum of two components (<u>signal/noise separation</u>):

- **modulating signal provided noiseless transmission**
- **noise component provided no modulation (unmodulated carrier transmitted)**

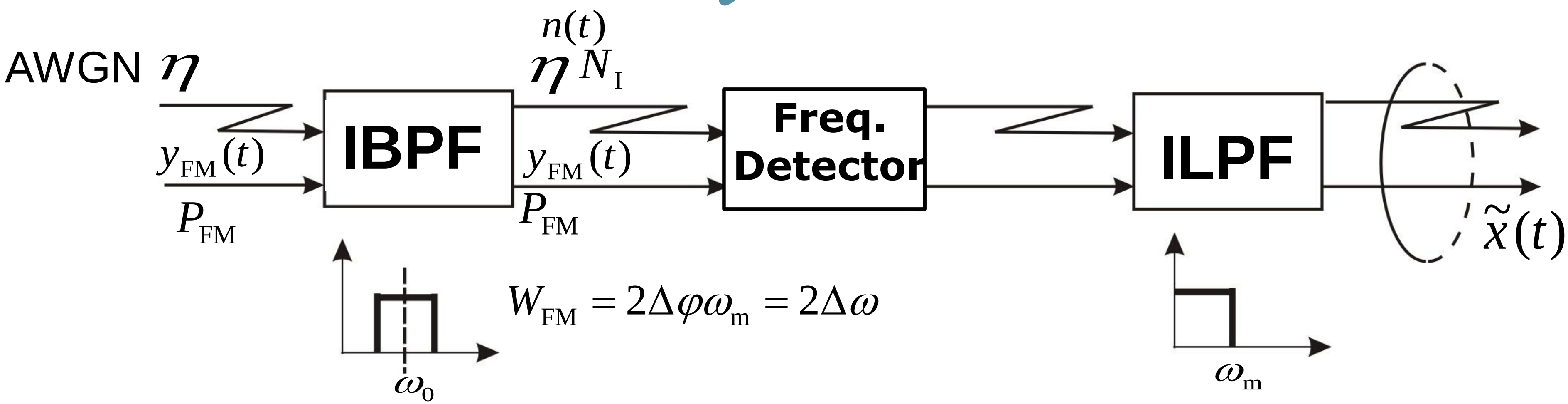
$$n_{\mathrm{I}}/A_0\,, n_{\mathrm{Q}}/A_0 \cong 0$$

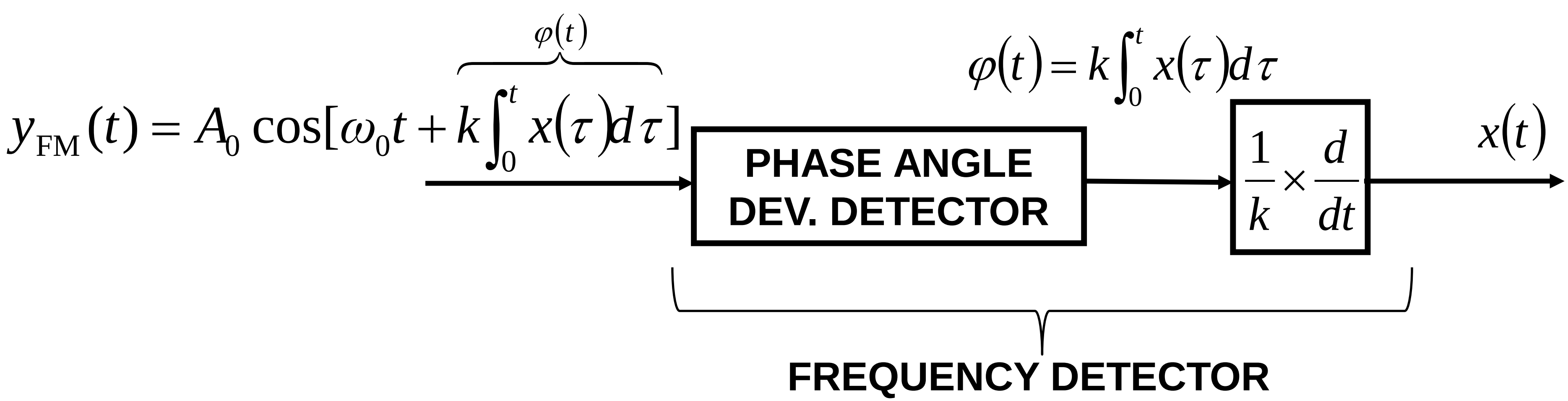
$$e(t) \cong e(t)\big|_{n=0} + e(t)\big|_{x=0}$$

$$e(t) \cong A_0[1+kx(t)] + A_0 + n_{\mathrm{I}}(t)$$

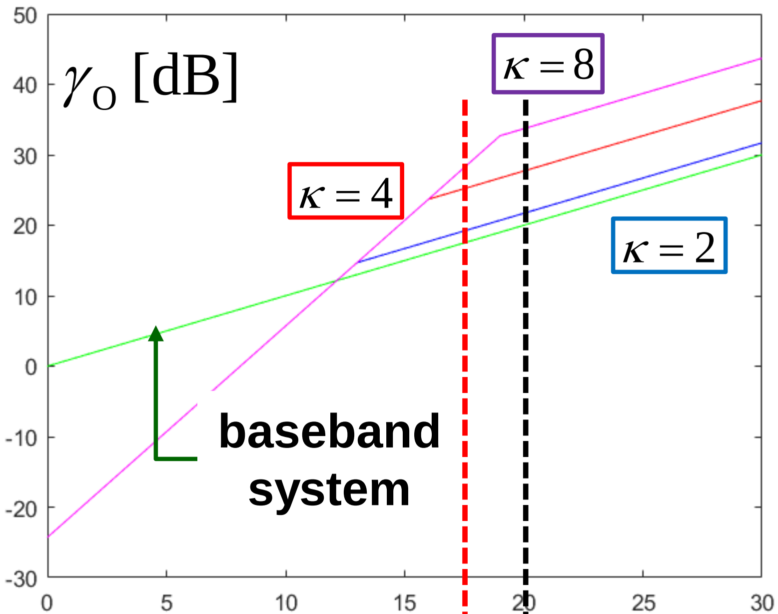
$$\tilde{x}(t) = \mathrm{FDP}[e(t)] \approx kA_0 x(t) + n_{\mathrm{I}}(t)$$

FM detection (by phase angle differentiation)





FM Noise Characteristic



$$g_{\mathrm{FM}} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}}\kappa} = \frac{3}{2}\Delta^2\varphi$$

Overthreshold operation

$$\gamma_{\mathrm{O}} \approx \frac{3}{8}\kappa^2\gamma$$

$$\gamma_{\mathrm{O}}[\mathrm{dB}] = -4{,}3 + 20\log\kappa + \gamma[\mathrm{dB}]$$

Underthreshold operation

$$\gamma_{\mathrm{O}}[\mathrm{dB}] \approx 3\gamma[\mathrm{dB}]$$

Threshold effect for $\gamma_{\mathrm{I}}^{*} = 10\,\mathrm{dB}$

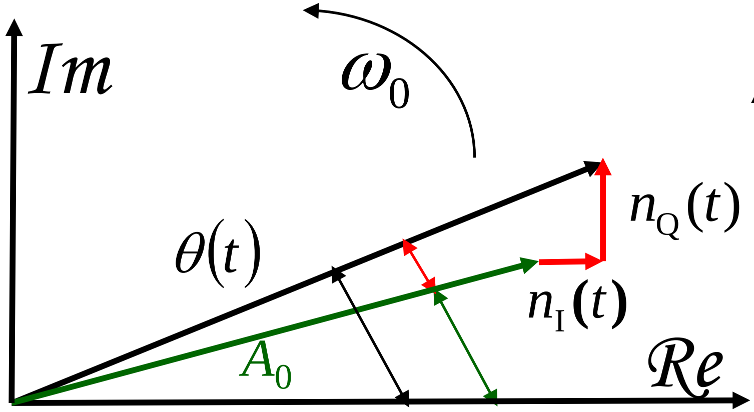
$\gamma = \kappa\gamma_{\mathrm{I}}$

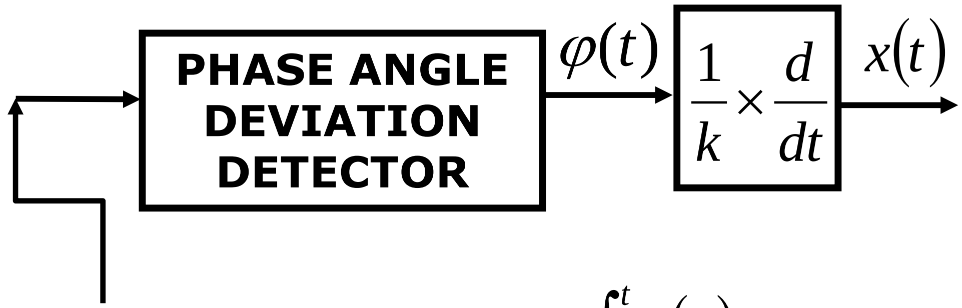
$\gamma^{*}[\mathrm{dB}] = 10\log\kappa + \gamma_{\mathrm{I}}^{*}[\mathrm{dB}] = 10\log\kappa + 10$

Extending the FM bandwidth results in a higher output *SNR* (*SNR* - bandwidth tradeoff).

The SNR – bandwidth tradeoff is limited by a threshold effect.

FM detection – overthreshold operation





$$y_{FM}(t) = A_0 \cos[\omega_0 t + \underbrace{k\int_0^t x(\tau)d\tau}_{\varphi(t)}]$$

$$\varphi(t) = k\int_0^t x(\tau)d\tau + \text{noise angle deviation}$$

For an overthreshold operation $n_I/A_0\,, n_Q/A_0 \approx 0$

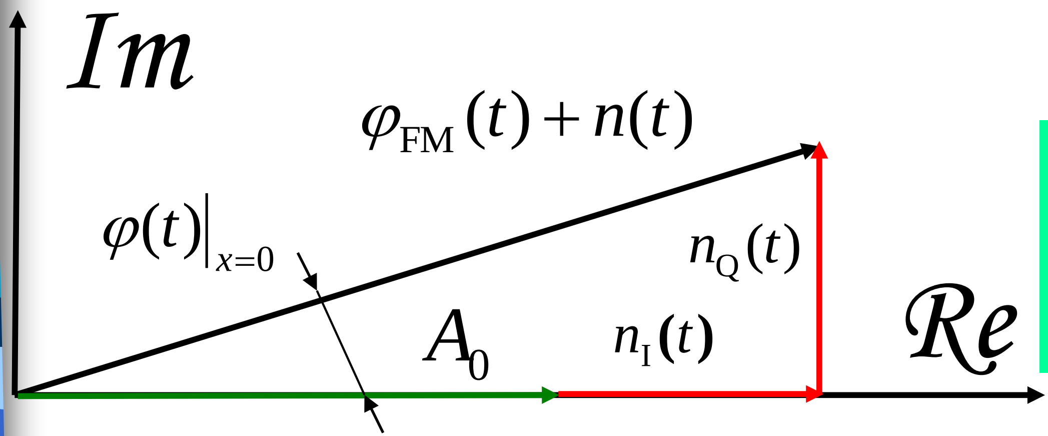
it may be proved that the FM detector fed by the FM signal corrupted by the ANGN $\varphi_{FM}(t) + n(t)$

produces on its output corrupted modulating signal: $x(t) + \dfrac{n'_Q(t)}{kA_0}$

$$x(t) + \frac{n'_Q(t)}{kA_0}$$

$$\tilde{x}(t) = x(t) + \text{noise}$$

FM detection – overthreshold operation (approach by signal/noise separation)



Overthreshold operation (signal/noise separation)

$$n_{\mathrm{I}}/A_0\,,\,n_{\mathrm{Q}}/A_0\approx 0$$

$$\varphi(t)\cong\varphi(t)\big|_{n=0}+\varphi(t)\big|_{x=0}$$

$$\mathrm{tg}\,\varphi(t)\big|_{x=0}=\frac{n_{\mathrm{Q}}(t)}{A_0+n_{\mathrm{I}}(t)}=\frac{n_{\mathrm{Q}}(t)/A_0}{1+n_{\mathrm{I}}(t)/A_0}\cong\frac{n_{\mathrm{Q}}(t)}{A_0}\cong\varphi(t)\big|_{x=0}$$

$$\varphi(t)\cong\varphi(t)\big|_{n=0}+\varphi(t)\big|_{x=0}=k\int_0^t x(\tau)d\tau+\frac{n_{\mathrm{Q}}(t)}{A_0}$$

$$\frac{1}{k}\varphi'(t)\cong x(t)+\frac{n'_{\mathrm{Q}}(t)}{kA_0}$$

$$\tilde{x}(t)=\mathrm{LPF}\left[x(t)+\frac{n'_{\mathrm{Q}}(t)}{kA_0}\right]=x(t)+\mathrm{LPF}\left[\frac{n'_{\mathrm{Q}}(t)}{kA_0}\right]$$

Modulation Gain – single tone FM (output SNR)

$$\tilde{x}(t) = \mathrm{LPF}[x(t) + n'_{\mathrm{Q}}(t)/kA_0]$$

$$\tilde{x}(t) = x(t) + \mathrm{LPF}\left[n'_{\mathrm{Q}}(t)/kA_0\right]$$

$$x(t) = a\sin\omega_{\mathrm{m}}t$$

$$\gamma_{\mathrm{O}} = \frac{P_{\mathrm{O}}}{N_{\mathrm{O}}} \qquad P_{\mathrm{O}} = \overline{x^2} = \frac{1}{2}a^2$$

$$N_{\mathrm{O}} = \overline{n_{\mathrm{O}}^{\ 2}} = \frac{1}{\pi k^2 A_0^2}\int_0^{\omega_{\mathrm{m}}} S_{n'_{\mathrm{Q}}}(\omega)d\omega = \frac{1}{\pi k^2 A_0^2}\int_0^{\omega_{\mathrm{m}}} 2\eta\omega^2 d\omega = \frac{2\eta\omega_{\mathrm{m}}^3}{3\pi k^2 A_0^2}$$

$$\gamma_{\mathrm{O}} = \frac{1}{2}a^2 \times \frac{3\pi k^2 A_0^2}{2\eta\omega_{\mathrm{m}}^3} = \frac{3\pi k^2 a^2 A_0^2}{4\eta\omega_{\mathrm{m}}^3} = \frac{3\pi\Delta^2\omega A_0^2}{4\eta\omega_{\mathrm{m}}^3} = \frac{3\pi\Delta^2\varphi A_0^2}{4\eta\omega_{\mathrm{m}}}$$

Modulation Gain – single tone FM (input SNR)

$$\gamma_{\mathrm{I}} = \frac{P_{\mathrm{I}}}{N_{\mathrm{I}}} = \frac{P_{\mathrm{FM}}}{\eta W_{\mathrm{FM}}/\pi} = \frac{A_0^2/2}{\eta W_{\mathrm{FM}}/\pi} = \frac{\pi A_0^2}{2\eta W_{\mathrm{FM}}} \qquad g_{\mathrm{FM}} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}}\kappa} = \frac{3\pi\Delta^2\varphi A_0^2}{4\eta\omega_{\mathrm{m}}} \times \frac{2\eta W_{\mathrm{FM}}}{\pi A_0^2} \times \frac{\omega_{\mathrm{m}}}{W_{\mathrm{FM}}}$$

$$g_{\mathrm{FM}} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}}\kappa} = \frac{3}{2}\Delta^2\varphi$$

$$\Delta\varphi = 2$$

$$g_{\mathrm{FM}} = 6 \approx 8\,\mathrm{dB}$$

FM noise characteristic

$$\gamma_{\mathrm{I}}\kappa = \gamma$$

$$\kappa = W_{\mathrm{FM}}/\omega_{\mathrm{m}} = 2\Delta\omega/\omega_{\mathrm{m}} = 2\Delta\varphi$$

$$\gamma_{\mathrm{O}} \approx \frac{3}{8}\kappa^2\gamma$$

$$\gamma_{\mathrm{O}}[\mathrm{dB}] = -4{,}3 + 2\times\kappa[\mathrm{dec}] + \gamma[\mathrm{dB}]$$

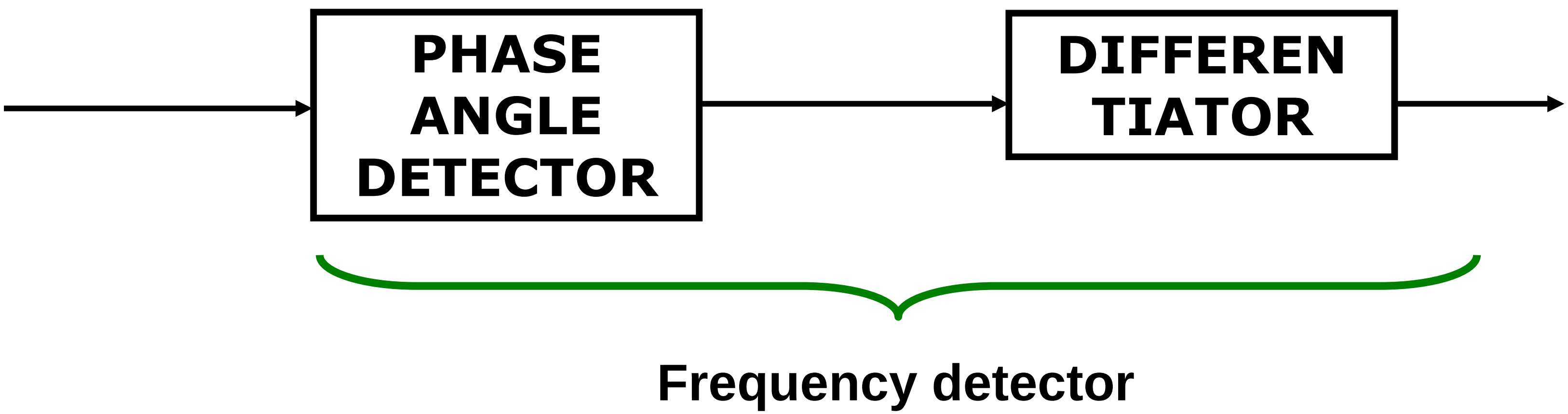
Bandwidth – *SNR* tradeoff

SNR improves by 6 dB for each 2:1 (3 dec) increase in a bandwidth.

FM Threshold Effect (phasor explanation)

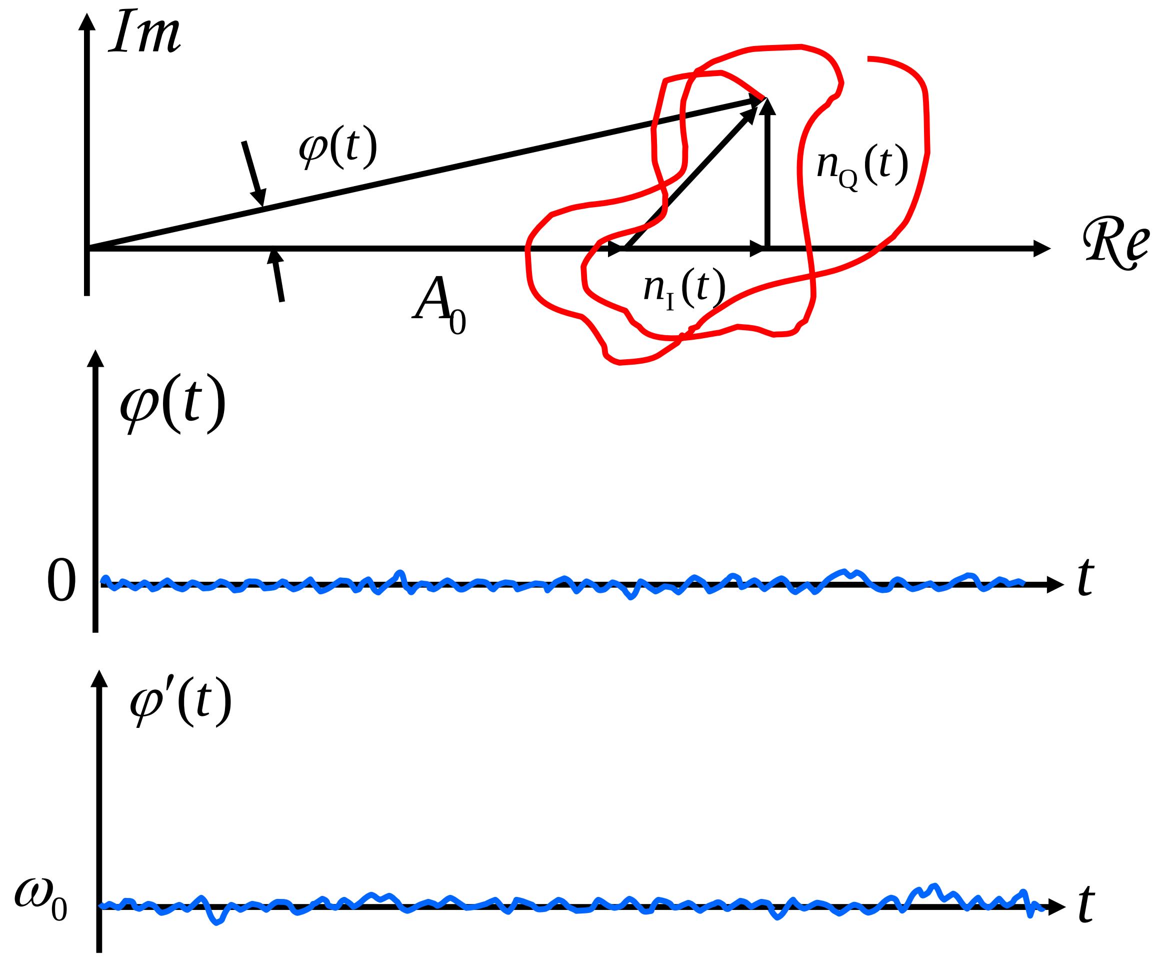
$$A_0\cos\omega_0 t + n_I(t)\cos\omega_0 t - n_Q(t)\sin\omega_0 t =$$

$$= \mathrm{Re}\left\{\left[A_0 + n_I(t) + jn_Q(t)\right]e^{j\omega_0 t}\right\}$$



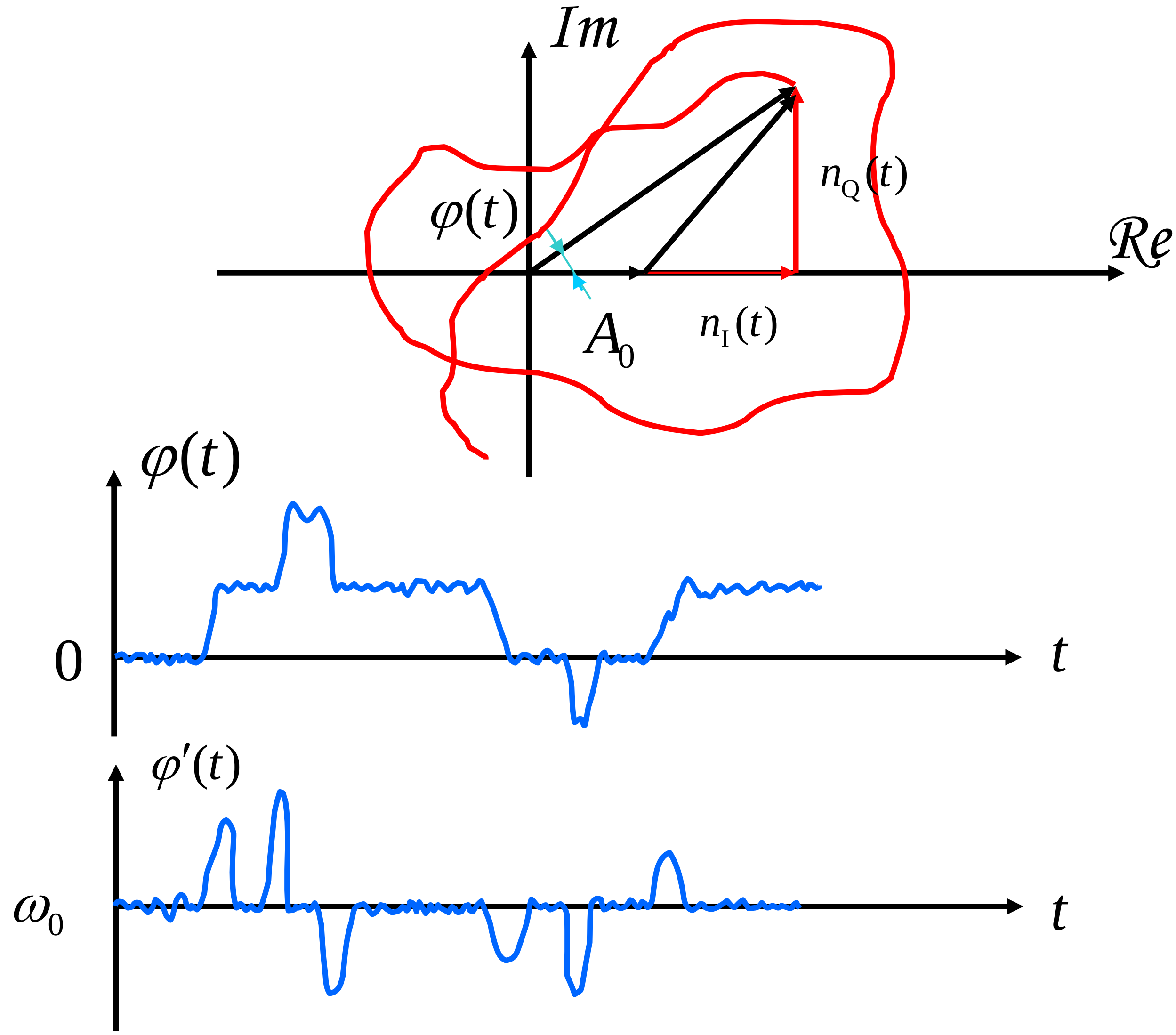
FM Threshold Effect (phasor explanation)

$$n_{\mathrm{I}}, n_{\mathrm{Q}} << A_0$$



FM Threshold Effect (phasor explanation)

$$n_I, n_Q >> A_0$$



Summary

- Immunity of a modulation to channel noise is expressed with a modulation gain (loss).
- Modulation gain is expressed as a ratio of an output SNR to an input SNR related to a system bandwidth.
- Noise characteristic (in a dB-dB scale) is a plot of an output SNR to an input SNR.
- Both AM and FM detection are nonlinear processess so their noise characteristics exhibit threshold effect.
- FM modulation makes possible to trade an output SNR to a system bandwidth. The trade-off is restricted by a threshold effect.